Comment on "Quantum bounce and cosmic recall"

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A recently derived inequality on volume fluctuations of a bouncing cosmology, valid for states which are semiclassical long after the bounce, does not restrict pre-bounce fluctuations sufficiently strongly to conclude that the pre-bounce state was semiclassical except in a very weak sense.

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The authors of [1] consider a loop quantum cosmological model for a free, massless scalar field ϕ, which is known to bounce without a big bang singularity. The question of interest is whether a generic state *assumed only to be semiclassical at late times* must have been semiclassical at early times. An inequality is derived for the difference of relative volume fluctuations *squared*,

$$D := \left| \lim_{\phi\to\infty} \left(\frac{(\Delta V)(\phi)^2}{\langle \hat{V}\rangle(\phi)^2} - \frac{(\Delta V)(-\phi)^2}{\langle \hat{V}\rangle(-\phi)^2} \right) \right| < 8\frac{\Delta b}{\langle \hat{b}\rangle} \quad (1)$$

where ϕ is a measure of time and b a curvature parameter conjugate to V. Eq. (1) shows that relative volume fluctuations can only change by a very small value in absolute terms. To specify this further, the authors consider a numerical example in which relative curvature fluctuations are of the size 10^{-57} as the amount by which squared volume fluctuations can change. From this observation, the authors state "(an almost) total recall."

Implications of (1) depend on the notion of semiclassicality used, which is only a very weak one in [1]. To analyze this properly, it is useful to ask what (1) tells us about the *relative* change in relative volume fluctuations. Absolutely, $\Delta V/\langle \hat{V}\rangle$ cannot change much. But the size of relative volume fluctuations is very small in the first place, and adding a small number such as 10^{-57} to something possibly even smaller could mean a significant change. Also the degree of smallness matters, not just smallness compared to one. Here, it becomes important that on the left hand side of (1) we have the relative fluctuations *squared*, but not on the right: For $\Delta b/\langle \hat{b}\rangle \sim \Delta V/\langle \hat{V}\rangle$, each term on the left of (1) is smaller than the right hand side, not just the difference.

In their example, the authors use the more specific assumption that, in the late-time semiclassical state, $\Delta V/\langle \hat{V}\rangle$ is nearly equal to $\Delta b/\langle \hat{b}\rangle$, with minimal uncertainty product ($\alpha_1 \sim 1$). This gives rise to a crucial inconsistency in [1] because the authors use here a condition much stronger than their understanding of semiclassicality elsewhere. At late times $\sqrt{G}\phi \gg 1$ we have $\Delta V/\langle \hat{V}\rangle \sim 10^{-57}$. The first term in D is then 10^{-114}, but D is bounded by the much larger 10^{-57}. If the inequality can be saturated, $\Delta V/\langle \hat{V}\rangle$ before the bounce, at $-\sqrt{G}\phi \gg 1$, could be as large as $\sqrt{10^{-57}} \approx 10^{-28}$ which is huge compared to the value at late times. The uncertainty product is then far from minimal ($\alpha_1 \sim 10^{28} \gg 1$) unlike at late times, which invalidates the conclusion of [1]: Reproducing a number up to a factor of $10^{\pm 28}$ hardly constitutes "(an almost) total recall."

After the bounce only semiclassicality is supposed to be assumed, but a minimum uncertainty product is actually used. There are only two consistent options to conclude: (i) the early state was *not semiclassical* because its uncertainty product can be much larger than minimal, or (ii) it is considered *semiclassical by weaker standards*. In the second case one has to justify the much stronger late-time assumption $\alpha_1 \sim 1$ — or relax it. A justification has not been provided; and if it is relaxed to allow deviations from minimal uncertainty by a factor up to $\alpha_1 \sim 10^{28}$, the early uncertainty product would be allowed to be even larger (by 10^{42}). Again, this is much weaker than the new late-time condition. Comparable pre- and post-bounce uncertainty products are implied by (1) only if $\Delta V/\langle \hat{V}\rangle \sim 1$, which is *not semiclassical*. The analysis in [1] regarding semiclassicality is intrinsically inconsistent due to different standards used, which shows that the role of semiclassicality of states is much more subtle than it may appear.

In [2], by contrast, dynamical coherent states provide bounds for the *relative* change of relative volume fluctuations, exploiting the availability of a solvable model [3, 4]. Coherence here means that the uncertainty relation is aleays exactly saturated. Thus, the assumption is stronger than semiclassicality, allowing more control. Still, relative changes even in such a state are bounded only by a factor of around 20. This is much smaller than the numbers controlled by (1), but still more than one as one should have it for what one could call a recall. This is the basis of cosmic forgetfulness [5]: not all the fluctuations (and higher moments) of a state before the bounce can be recovered after the bounce, and values depend very sensitively on the late-time state.

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